Module 2

DC Circuit

Lesson 8

Thevenin's and Norton's theorems in the context of dc voltage and current sources acting in a resistive network

Objectives

- To understand the basic philosophy behind the Thevenin's theorem and its application to solve dc circuits.
- Explain the advantage of Thevenin's theorem over conventional circuit reduction techniques in situations where load changes.
- Maximum power transfer theorem and power transfer efficiency.
- Use Norton's theorem for analysis of dc circuits and study the advantage of this theorem over conventional circuit reduction techniques in situations where load changes.

L.8.1 Introduction

A simple circuit as shown in fig.8.1 is considered to illustrate the concept of equivalent circuit and it is always possible to view even a very complicated circuit in terms of much simpler equivalent source and load circuits. Subsequently the reduction of computational complexity that involves in solving the current through a branch for different values of load resistance (R_L) is also discussed. In many applications, a network may contain a variable component or element while other elements in the circuit are kept constant. If the solution for current (I) or voltage (V) or power (P) in any component of network is desired, in such cases the whole circuit need to be analyzed each time with the change in component value. In order to avoid such repeated computation, it is desirable to introduce a method that will not have to be repeated for each value of variable component. Such tedious computation burden can be avoided provided the fixed part of such networks could be converted into a very simple equivalent circuit that represents either in the form of practical voltage source known as Thevenin's voltage source ($V_{Th} = magnitude\ of\ voltage\ source$, $R_{Th} = internal\ resistance\ of\ the\ source$) or in the form of practical current source known as Norton's current source ($I_N = magnitude\ of\ current\ source$, $R_N = internal\ resistance\ of\ current\ source$). In true sense, this conversion will considerably simplify the analysis while the load resistance changes. Although the conversion technique accomplishes the same goal, it has certain advantages over the techniques that we have learnt in earlier lessons.

Let us consider the circuit shown in fig. 8.1(a). Our problem is to find a current through R_L using different techniques; the following observations are made.

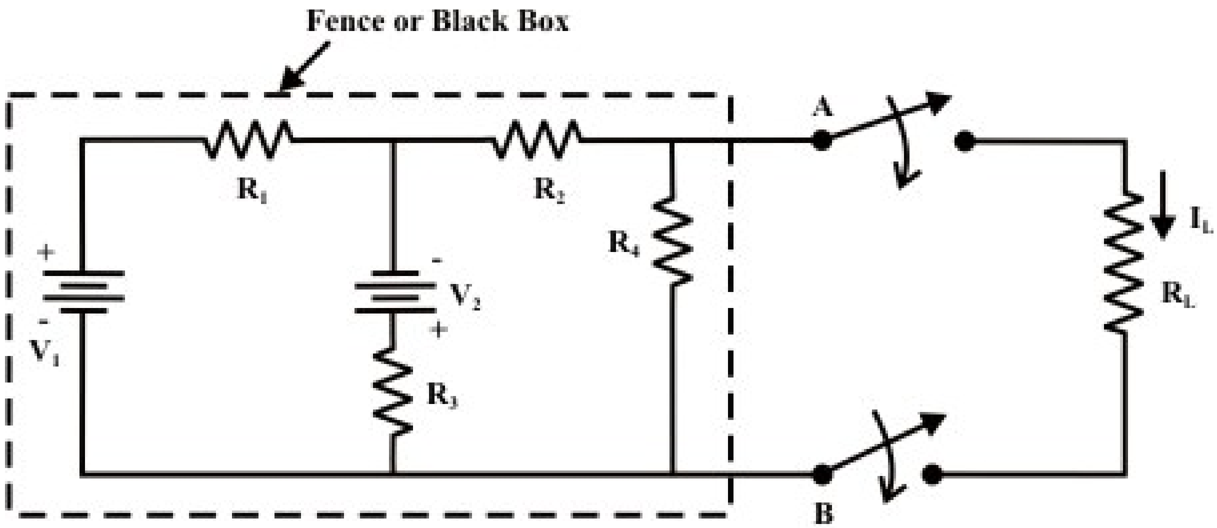


Fig. 8.1(a): A simple dc network

Find

- Mesh current method needs 3 equations to be solved
- Node voltage method requires 2 equations to be solved
- Superposition method requires a complete solution through load resistance (R_L) by considering each independent source at a time and replacing other sources by their internal source resistances.

Suppose, if the value of R_L is changed then the three (mesh current method) or two equations (node voltage method) need to be solved again to find the new current in R_L. Similarly, in case of superposition theorem each time the load resistance R_L is changed, the entire circuit has to be analyzed all over again. Much of the tedious mathematical work can be avoided if the fixed part of circuit (fig. 8.1(a)) or in other words, the circuit contained inside the imaginary fence or black box with two terminals A & B, is replaced by the simple equivalent voltage source (as shown in fig. 8.1(b)) or current source (as shown in fig.8.1(c)).

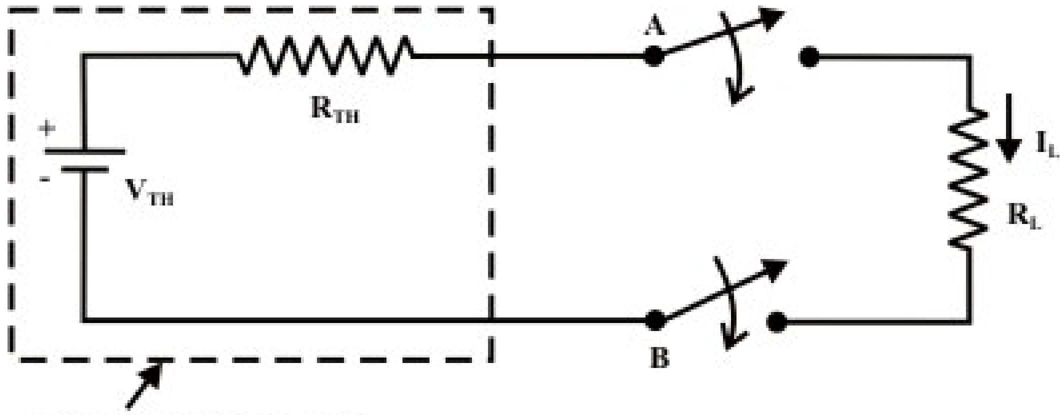


Fig. 8.1(b): circuit 8.1(a) is equivalently replaced by a simple practical voltage source

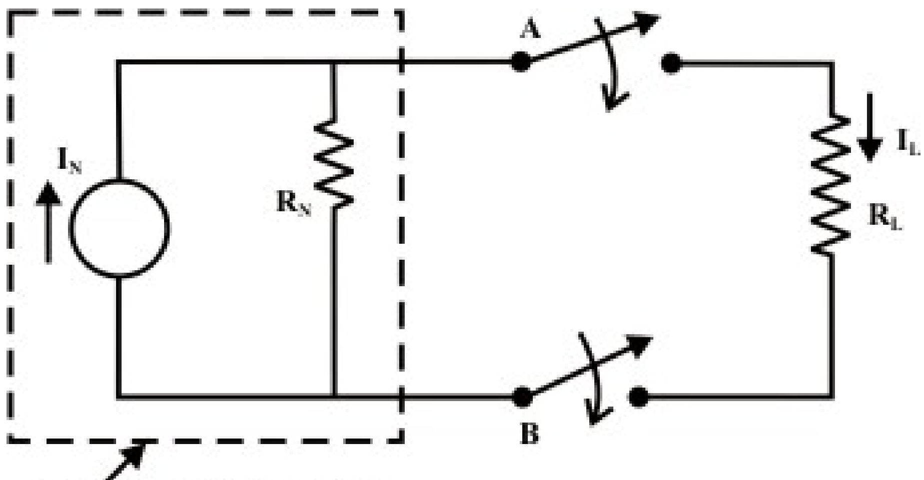


Fig. 8.1(c): Circuit 8.1(a) is equivalently replaced by a simple practical current source

Thevenin's Theorem: Thevenin's theorem states that any two output terminals (A & B , shown in fig. 8.2.(a)) of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a simple voltage source of magnitude V_{Th} in series with a single resistor R_{Th} (see fig. 8.2(d)) where R_{Th} is the equivalent resistance of the network when looking from the output terminals A & B with all sources (voltage and current) removed and replaced by their internal resistances (see fig. 8.2(c)) and the magnitude of V_{Th} is equal to the open circuit voltage across the A & B terminals. (The proof of the theorem will be given in section- L8. 5).

L.8.2 The procedure for applying Thevenin's theorem

To find a current I_L through the load resistance R_L (as shown in fig. 8.2(a)) using Thevenin's theorem, the following steps are followed:

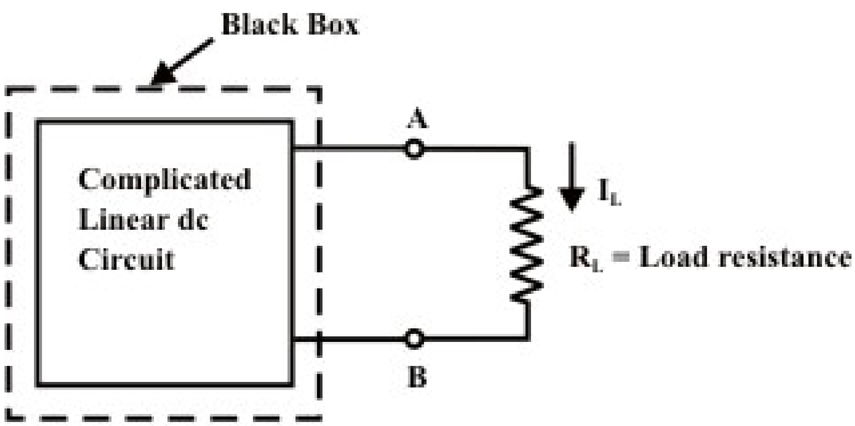


Fig. 8.2(a)

Step-1: Disconnect the load resistance (R_L) from the circuit, as indicated in fig. 8.2(b).

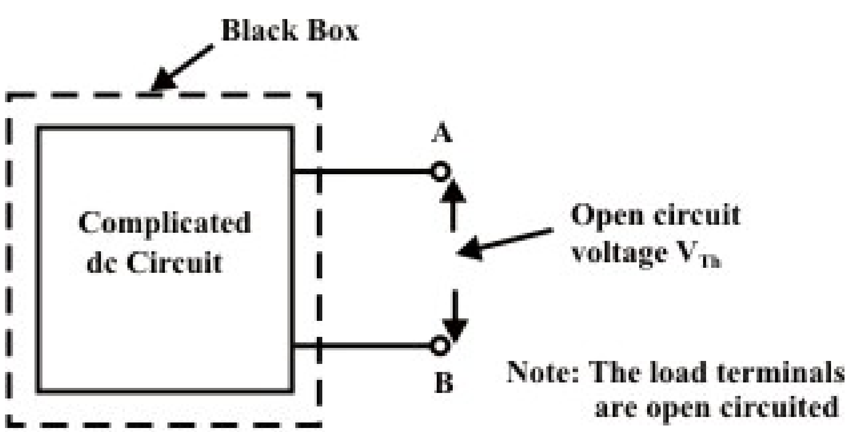


Fig. 8.2(b)

Step-2: Calculate the open-circuit voltage V_{TH} (shown in fig.8.2(b)) at the load terminals ($A \& B$) after disconnecting the load resistance (R_L). In general, one can apply any of the techniques (mesh-current, node-voltage and superposition method) learnt in earlier lessons to compute V_{Th} (experimentally just measure the voltage across the load terminals using a voltmeter).

Step-3: Redraw the circuit (fig. 8.2(b)) with each practical source replaced by its internal resistance as shown in fig.8.2(c). (note, voltage sources should be short-circuited (just remove them and replace with plain wire) and current sources should be open-circuited (just removed).

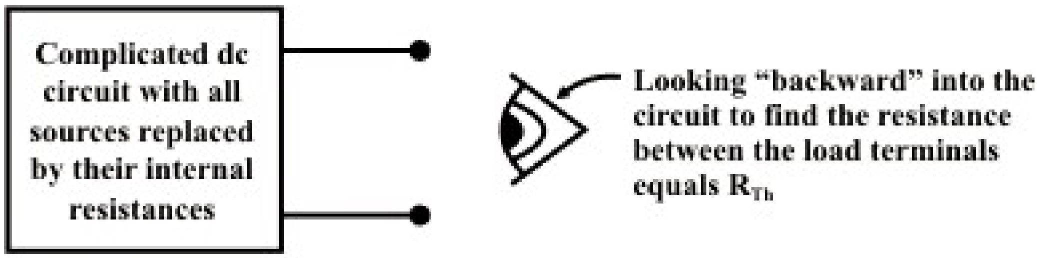


Fig. 8.2(c)

Step-4: Look backward into the resulting circuit from the load terminals ($A \& B$), as suggested by the eye in fig.L.8.2(c). Calculate the resistance that would exist between the load terminals (or equivalently one can think as if a voltage source is applied across the load terminals and then trace the current distribution through the circuit (fig.8.2 (c)) in order to calculate the resistance across the load terminals.) The resistance R_{Th} is described in the statement of Thevenin's theorem. Once again, calculating this resistance may be a difficult task but one can try to use the standard circuit reduction technique or $Y-\Delta$ *or* $\Delta-Y$ transformation techniques.

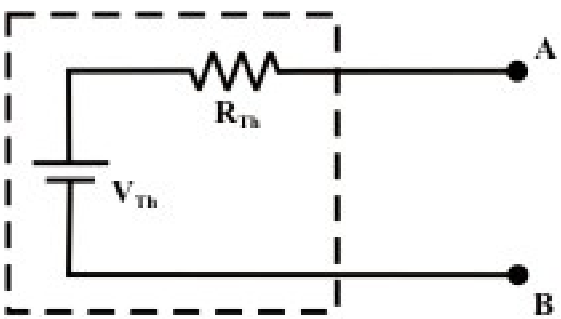


Fig. 8.2(d): Dash-portion of the circuit (Fig. 8.2(a)) is equivalently replaced by a practical voltage source.

Step-5: Place R_{Th} in series with V_{Th} to form the Thevenin's equivalent circuit (replacing the imaginary fencing portion or fixed part of the circuit with an equivalent practical voltage source) as shown in fig. 8.2(d).

Step-6: Reconnect the original load to the Thevenin voltage circuit as shown in fig.8.2(e); the load's voltage, current and power may be calculated by a simple arithmetic operation only.

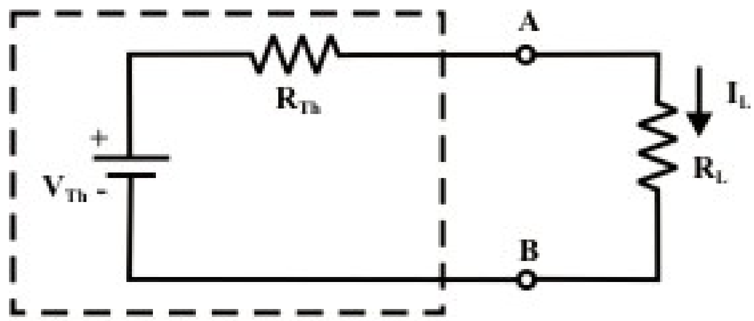


Fig. 8.2(e)

Load current $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$ (8.1)

Voltage across the load $V_L = \dfrac{V_{Th}}{R_{Th} + R_L} \times R_L = I_L \times R_L$ (8.2)

Power absorbed by the load $P_L = I_L^{\,2} \times R_L$ (8.3)

Remarks: (i) One great advantage of Thevenin's theorem over the normal circuit reduction technique or any other technique is this: once the Thevenin equivalent circuit has been formed, it can be reused in calculating load current (I_L), load voltage (V_L) and load power (P_L) for different loads using the equations (8.1)-(8.3).

(ii) Fortunately, with help of this theorem one can find the choice of load resistance R_L that results in the maximum power transfer to the load. On the other hand, the effort necessary to solve this problem-using node or mesh analysis methods can be quite complex and tedious from computational point of view.

L.8.3 Application of Thevenin's theorem

Example: L.8.1 For the circuit shown in fig.8.3(a), find the current through $R_L = R_2 = 1\Omega$ resistor (I_{a-b} branch) using Thevenin's theorem & hence calculate the voltage across the current source (V_{cg}).

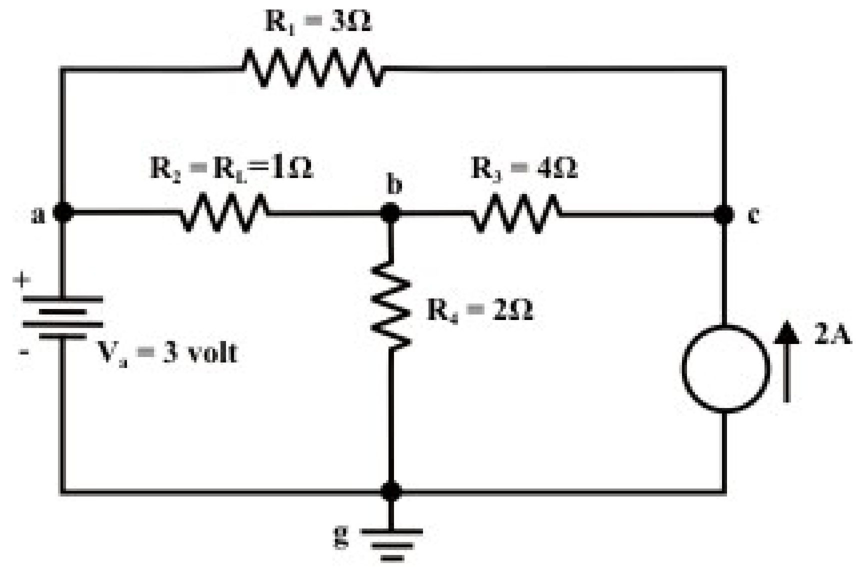


Fig. 8.3(a)

Solution:
Step-1: Disconnect the load resistance R_L and redraw the circuit as shown in fig.8.3(b).

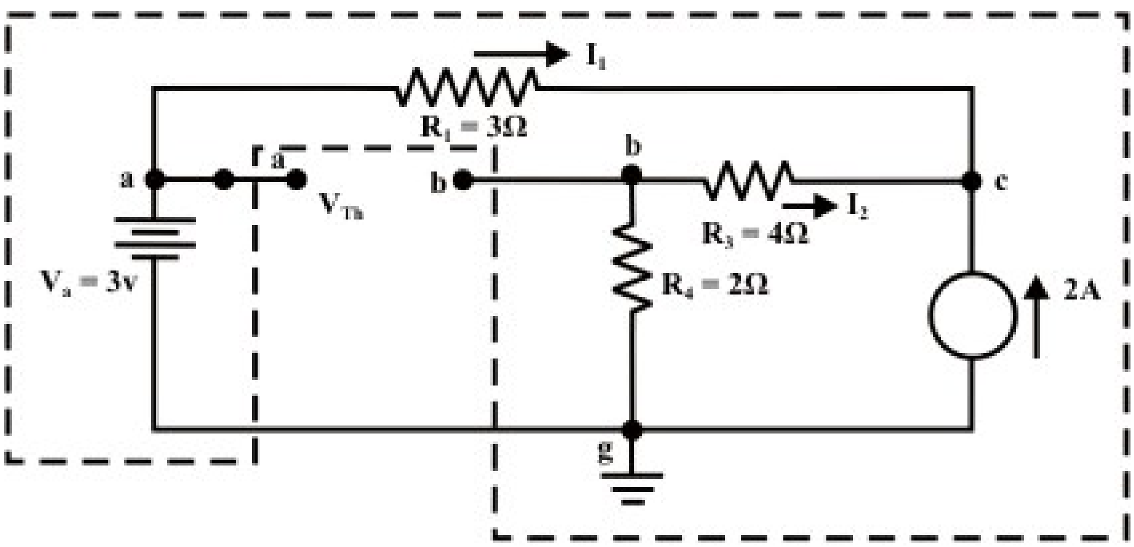


Fig. 8.3(b)

Step-2: Apply any method (say node-voltage method) to calculate V_{Th}.

At node C:

$$2 + I_1 + I_2 = 0$$

$$2 + \frac{(3 - V_c)}{3} + \frac{(0 - V_c)}{6} \Rightarrow V_c = 6\,volt$$

Now, the currents I_1 & I_2 can easily be computed using the following expressions.

$I_1 = \frac{V_a - V_c}{3} = \frac{3-6}{3} = -1A$ (note, current I_1 is flowing from 'c' to 'a')

$I_2 = \frac{0 - V_c}{6} = \frac{-6}{6} = -1A$ (note, current I_2 is flowing from 'c' to 'g')

Step-3: Redraw the circuit (fig.8.3(b) indicating the direction of currents in different branches. One can find the Thevenin's voltage V_{Th} using KVL around the closed path 'gabg' (see fig.8.3.(c).

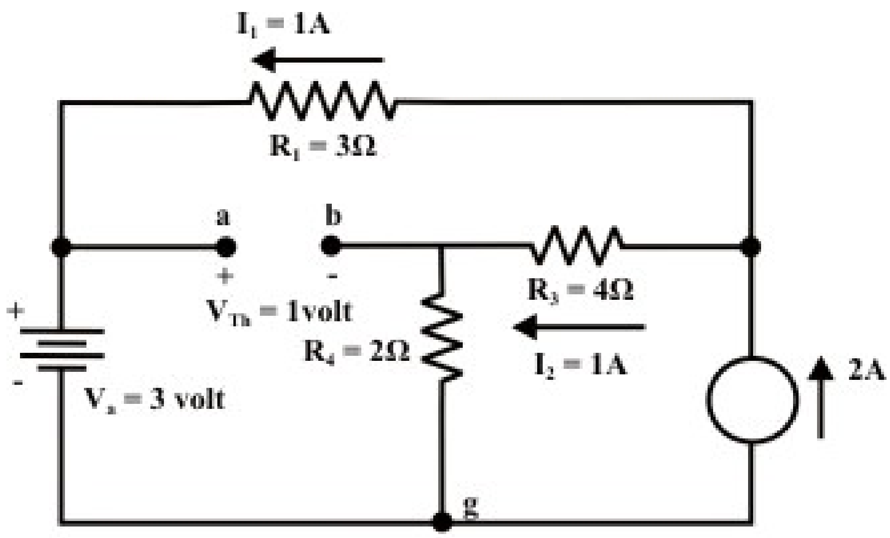


Fig. 8.3(c)

$V_{Th} = V_{ag} - V_{bg} = 3 - 2 = 1 volt$

Step-4: Replace all sources by their internal resistances. In this problem, voltage source has an internal resistance zero (0) (ideal voltage source) and it is short-circuited with a wire. On the other hand, the current source has an infinite internal resistance (ideal current source) and it is open-circuited (just remove the current source). Thevenin's resistance R_{Th} of the fixed part of the circuit can be computed by looking at the load terminals 'a'- 'b' (see fig.8.3(d)).

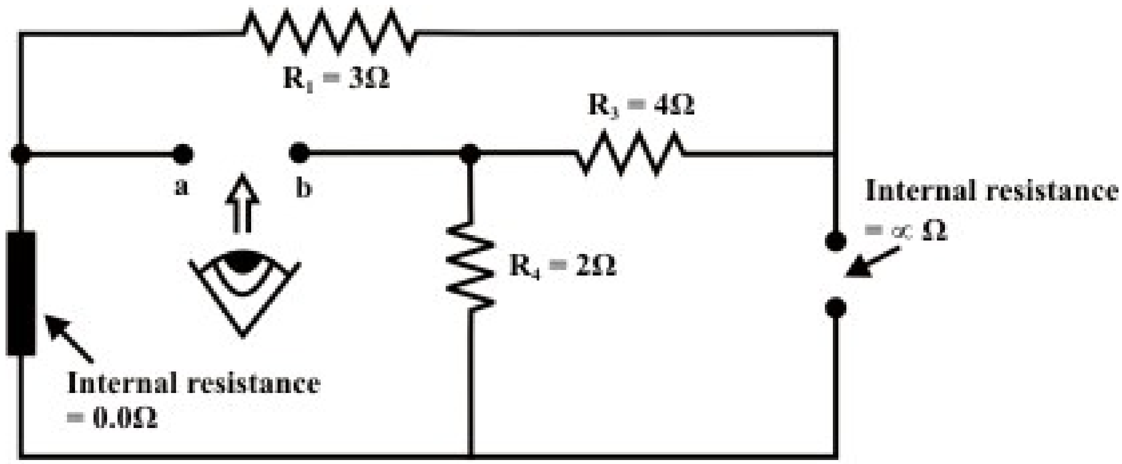


Fig. 8.3(d)

$$R_{Th} = (R_1 + R_3) || R_4 = (3+4) || 2 = 1.555\Omega$$

Step-5: Place R_{Th} in series with V_{Th} to form the Thevenin's equivalent circuit (a simple practical voltage source). Reconnect the original load resistance $R_L = R_2 = 1\Omega$ to the Thevenin's equivalent circuit (note the polarity of 'a' and 'b' is to be considered carefully) as shown in fig.8.3(e).

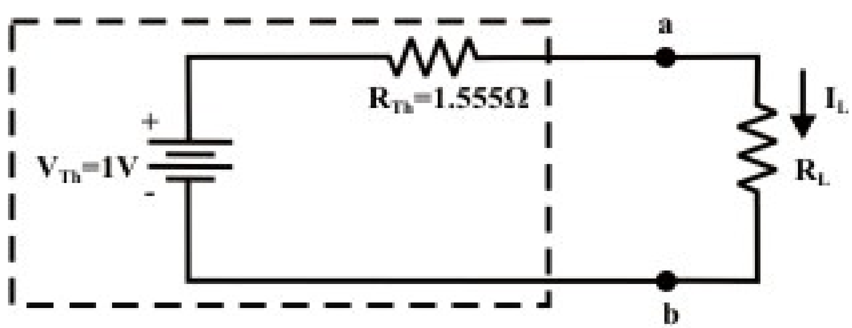


Fig. 8.3(e): Equivalent dc circuit fig. 8.3(b) is replace by a practical voltage source.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1}{1.555 + 1} = 0.39\, A (a \text{ to } b)$$

Step-6: The circuit shown in fig.8.3 (a) is redrawn to indicate different branch currents. Referring to fig.8.3 (f), one can calculate the voltage V_{bg} and voltage across the current source (V_{cg}) using the following equations.

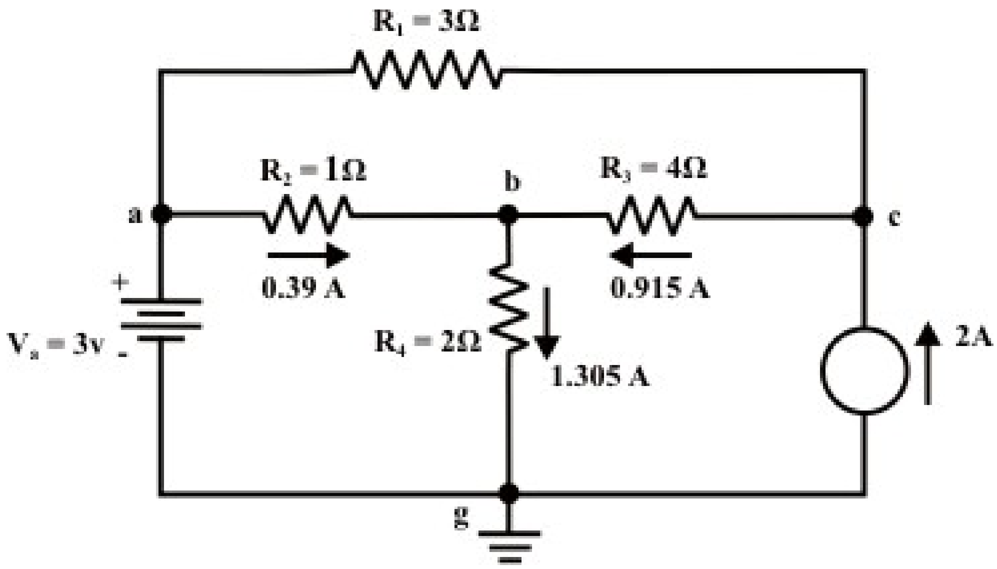


Fig. 8.3(f)

$V_{bg} = V_{ag} - V_{ab} = 3 - 1 \times 0.39 = 2.61 volt.$

$I_{bg} = \frac{2.61}{2} = 1.305\,A;\ I_{cb} = 1.305 - 0.39 = 0.915\,A$

$V_{cg} = 4 \times 0.915 + 2 \times 1.305 = 6.27\,volt.$

Example-L.8.2 For the circuit shown in fig.8.4 (a), find the current I_L through 6 Ω resistor using Thevenin's theorem.

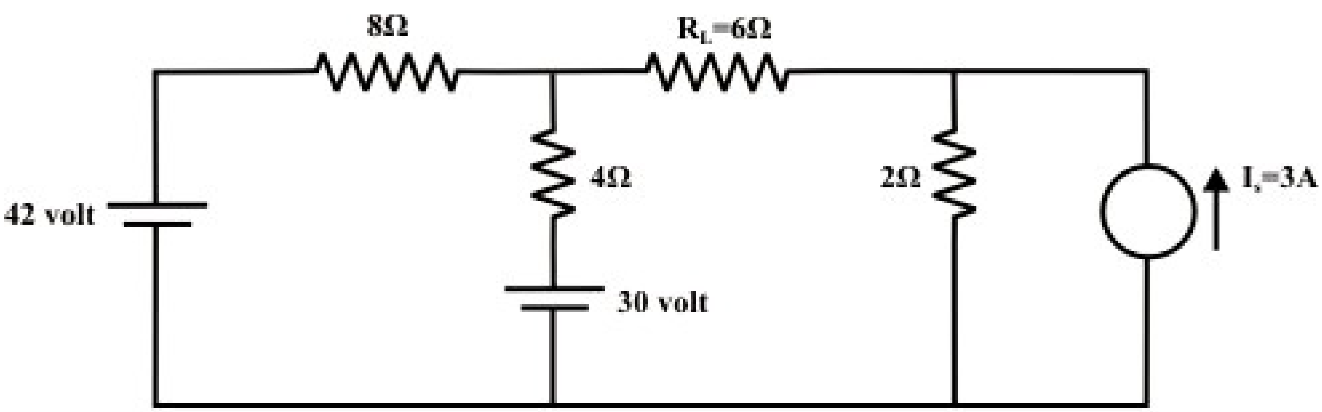


Fig. 8.4(a)

Solution:

Step-1: Disconnect 6Ω from the terminals 'a' and 'b' and the corresponding circuit diagram is shown in fig.L.8.4 (b). Consider point 'g' as ground potential and other voltages are measured with respect to this point.

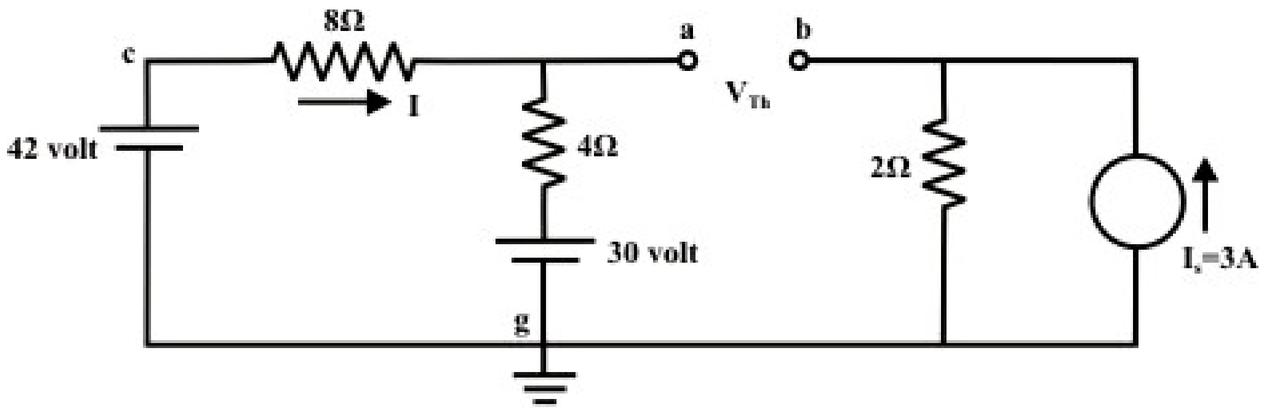


Fig. 8.4(b)

Step-2: Apply any suitable method to find the Thevenin's voltage (V_{Th}) (or potential between the terminals 'a' and 'b'). KVL is applied around the closed path 'gcag' to compute Thevenin's voltage.

$$42 - 8I - 4I - 30 = 0 \;\Rightarrow\; I = 1A$$
$$Now,\; V_{ag} = 30 + 4 = 34\,volt;\; V_{bg} = 2 \times 3 = 6\,volt.$$

$V_{Th} = V_{ab} = V_{ag} - V_{bg} = 34 - 6 = 28\,volt$ (note 'a' is higher potential than 'b')

Step-3: Thevenin's resistance R_{Th} can be found by replacing all sources by their internal resistances (all voltage sources are short-circuited and current sources are just removed or open circuited) as shown in fig.8.4 (c).

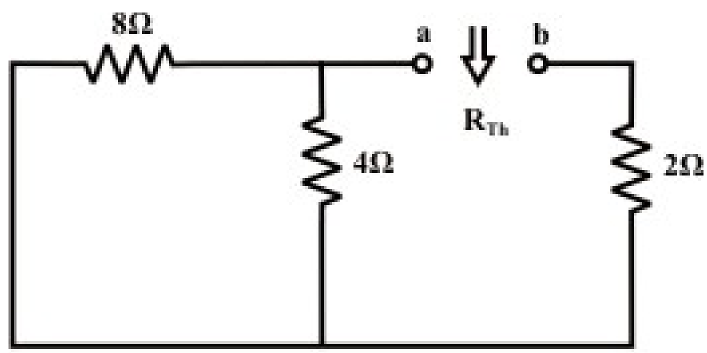


Fig. 8.4(c)

$$R_{Th} = (8 \,||\, 4) + 2 = \frac{8 \times 4}{12} + 2 = \frac{14}{3} = 4.666\,\Omega$$

Step-4: Thevenin's equivalent circuit as shown in fig.8.4 (d) is now equivalently represents the original circuit (fig.L.8.4(a).

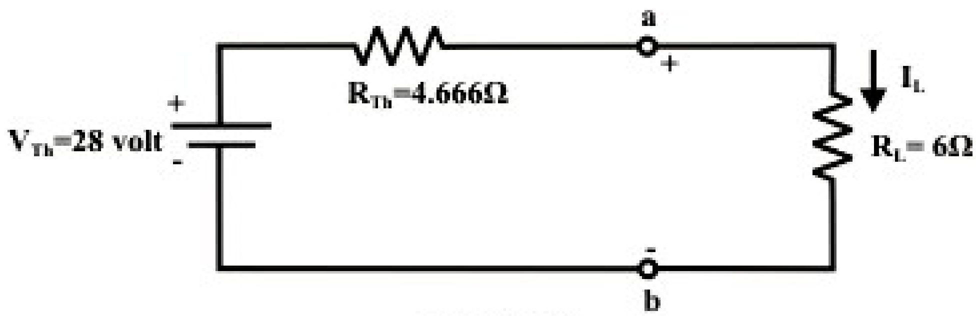


Fig. 8.4(d)

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{28}{4.666 + 6} = 2.625\,A$$

Example-L.8.3 The box shown in fig.8.5 (a) consists of independent dc sources and resistances. Measurements are taken by connecting an ammeter in series with the resistor R and the results are shown in table.

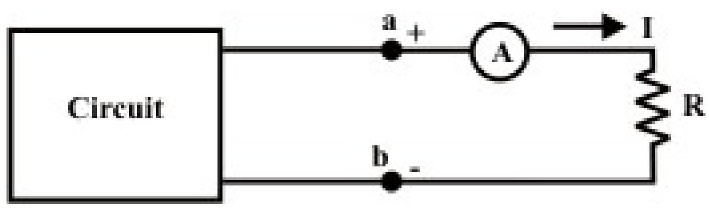


Fig. 8.5(a)

Table

R	I
10Ω	$2\,A$
20Ω	$1.5\,A$
?	$0.6\,A$

Solution: The circuit shown in fig.8.5(a) can be replaced by an equivalent Thevenin's voltage source as shown in fig.8.5(b). The current flowing through the resistor R is expressed as

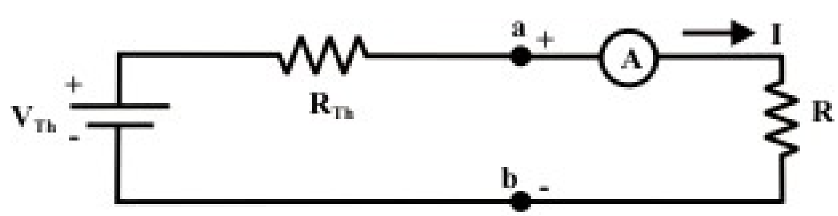


Fig. 8.5(b): Equivalent circuit of fig. 8.5(a)

$$I = \frac{V_{Th}}{R_{Th} + R} \tag{8.4}$$

The following two equations are written from measurements recorded in table.

$$\frac{V_{Th}}{R_{Th}+10}=2 \Rightarrow \; V_{Th}-2\,R_{Th}=20 \tag{8.5}$$

$$\frac{V_{Th}}{R_{Th}+20}=1.5 \Rightarrow \; V_{Th}-1.5\,R_{Th}=30 \tag{8.6}$$

Solving equations (8.5) and (8.6) we get,
$V_{Th}=60\ volt;\ R_{Th}=20\,\Omega$
The choice of R that yields current flowing the resistor is $0.6\,A$ can be obtained using the equation (8.4).

$$I=\frac{V_{Th}}{R_{Th}+R}=\frac{60}{20+R}=0.6 \;\Rightarrow\; R=80\,\Omega.$$

L.8.4 Maximum Power Transfer Theorem

In an electric circuit, the load receives electric energy via the supply sources and converts that energy into a useful form. The maximum allowable power receives by the load is always limited either by the heating effect (incase of resistive load) or by the other power conversion taking place in the load. The Thevenin and Norton models imply that the internal circuits within the source will necessarily dissipate some of power generated by the source. A logical question will arise in mind, how much power can be transferred to the load from the source under the most practical conditions? In other words, what is the value of load resistance that will absorbs the maximum power from the source? This is an important issue in many practical problems and it is discussed with a suitable example.

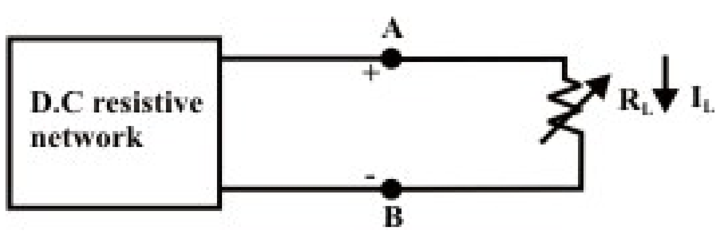


Fig. 8.6(a)

Let us consider an electric network as shown in fig.8.6(a), the problem is to find the choice of the resistance R_L so that the network delivers maximum power to the load or in other words what value of load resistance R_L will absorb the maximum amount of power from the network. This problem can be solved using nodal or mesh current analysis to obtain an expression for the power absorbed by R_L, then the derivative of this expression with respect to R_L will establish the condition under what circumstances the maximum power transfer occurs. The effort required for such an approach can be quite tedious and complex. Fortunately, the network shown in fig.L.8.6(a) can be represented by an equivalent Thevenin's voltage source as shown in fig.L.8.6(b).

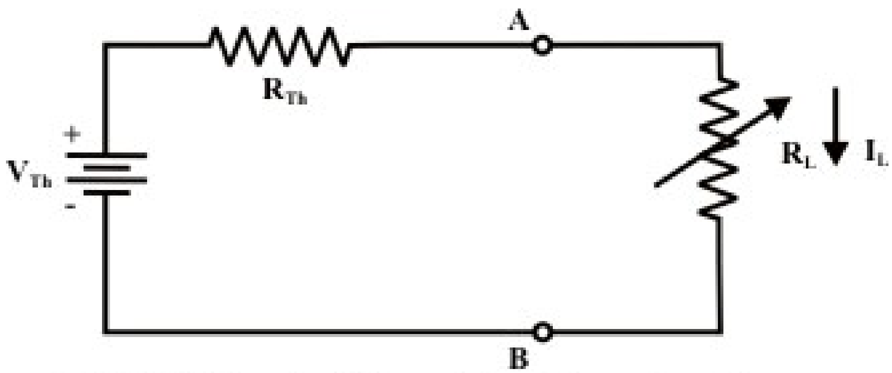


Fig. 8.6(b): The circuit for maximum Power transfer

In fig.8.6(b) a variable load resistance R_L is connected to an equivalent Thevenin circuit of original circuit(fig.8.6(a)). The current for any value of load resistance is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Then, the power delivered to the load is

$$P_L = I_L^{\,2} \times R_L = \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L$$

The load power depends on both R_{Th} *and* R_L; however, R_{Th} is constant for the equivalent Thevenin network. So power delivered by the equivalent Thevenin network to the load resistor is entirely depends on the value of R_L. To find the value of R_L that absorbs a maximum power from the Thevenin circuit, we differentiate P_L with respect to R_L.

$$\frac{dP(R_L)}{dR_L} = V_{Th}^{\;2} \left[\frac{(R_{Th} + R_L)^2 - 2R_L \times (R_{Th} + R_L)}{\left(R_{Th} + R_L\right)^4} \right] = 0 \Rightarrow (R_{Th} + R_L) - 2R_L = 0 \Rightarrow R_L = R_{Th} \tag{8.7}$$

For maximum power dissipation in the load, the condition given below must be satisfied

$$\left. \frac{d^2 P(R_L)}{dR_L^{\;2}} \right|_{R_L = R_{Th}} = -\frac{V_{Th}^{\;2}}{8R_{Th}} < 0$$

This result is known as "Matching the load" or maximum power transfer occurs when the load resistance R_L matches the Thevenin's resistance R_{Th} of a given systems. Also, notice that under the condition of maximum power transfer, the load voltage is, by voltage division, one-half of the Thevenin voltage. The expression for maximum power dissipated to the load resistance is given by

$$P_{\max} = \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L \Bigg|_{R_L = R_{Th}} = \frac{V_{Th}^{\ 2}}{4 R_{Th}}$$

The total power delivered by the source
$P_T = I_L^{\ 2} \left(R_{Th} + R_L \right) = 2 \times I_L^{\ 2} R_L$
This means that the Thevenin voltage source itself dissipates as much power in its internal resistance R_{Th} as the power absorbed by the load R_L. Efficiency under maximum power transfer condition is given by

$$Efficiency = \frac{I_L^{\ 2} R_L}{2 I_L^{\ 2} R_L} \times 100 = 50\% \qquad (8.8)$$

For a given circuit, V_{Th} *and* R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as shown in fig.8.6(c).

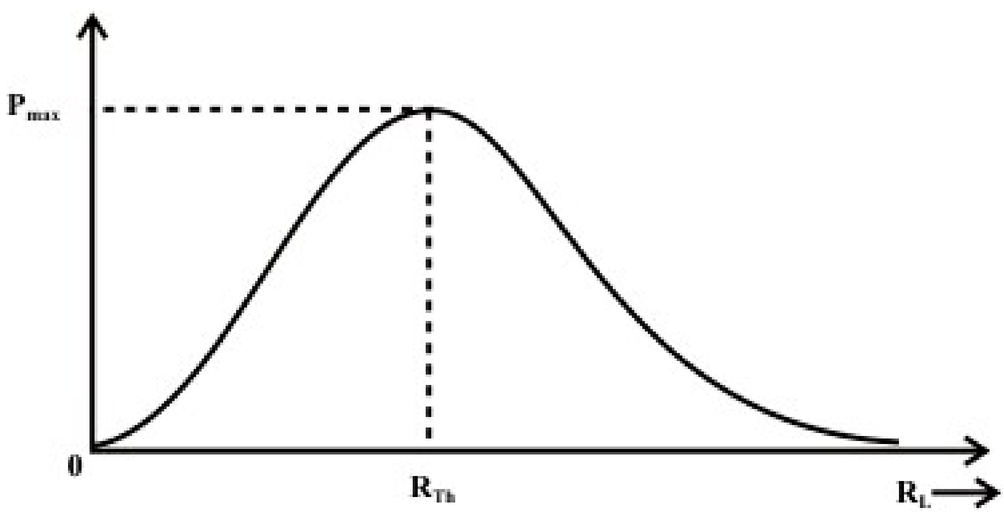


Fig. 8.6(c): Power dissipated to the load as a function of R_L

Remarks: The Thevenin equivalent circuit is useful in finding the maximum power that a linear circuit can deliver to a load.

Example-L.8.4 For the circuit shown in fig.8.7(a), find the value of R_L that absorbs maximum power from the circuit and the corresponding power under this condition.

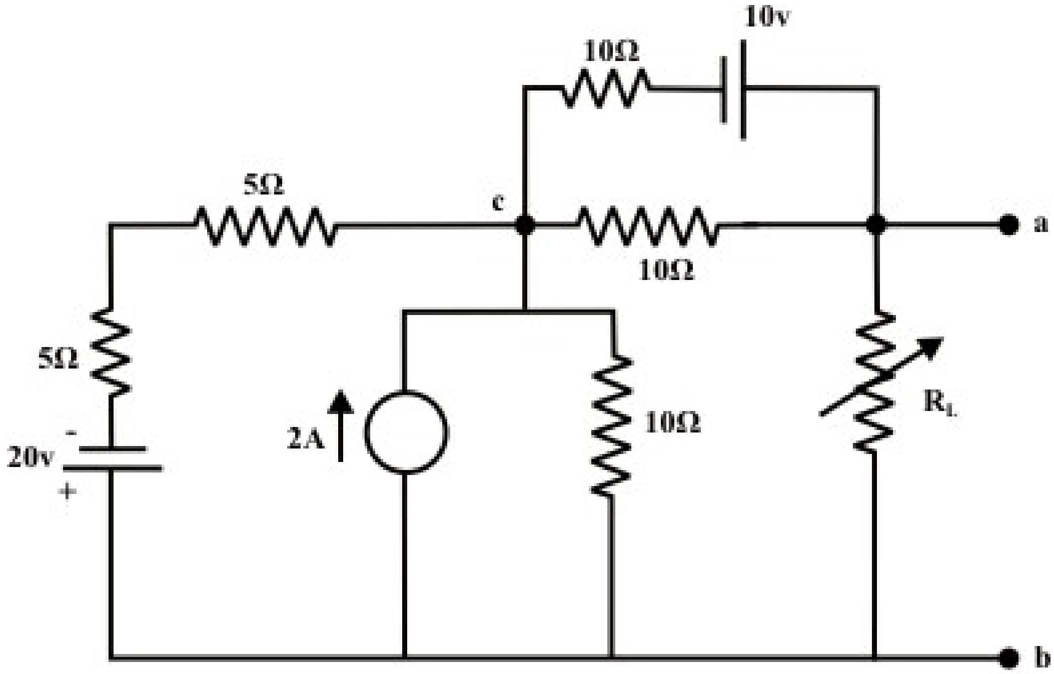


Fig. 8.7(a)

Solution: Load resistance R_L is disconnected from the terminals 'a' and 'b' and the corresponding circuit diagram is drawn (see fig.8.7(b)).

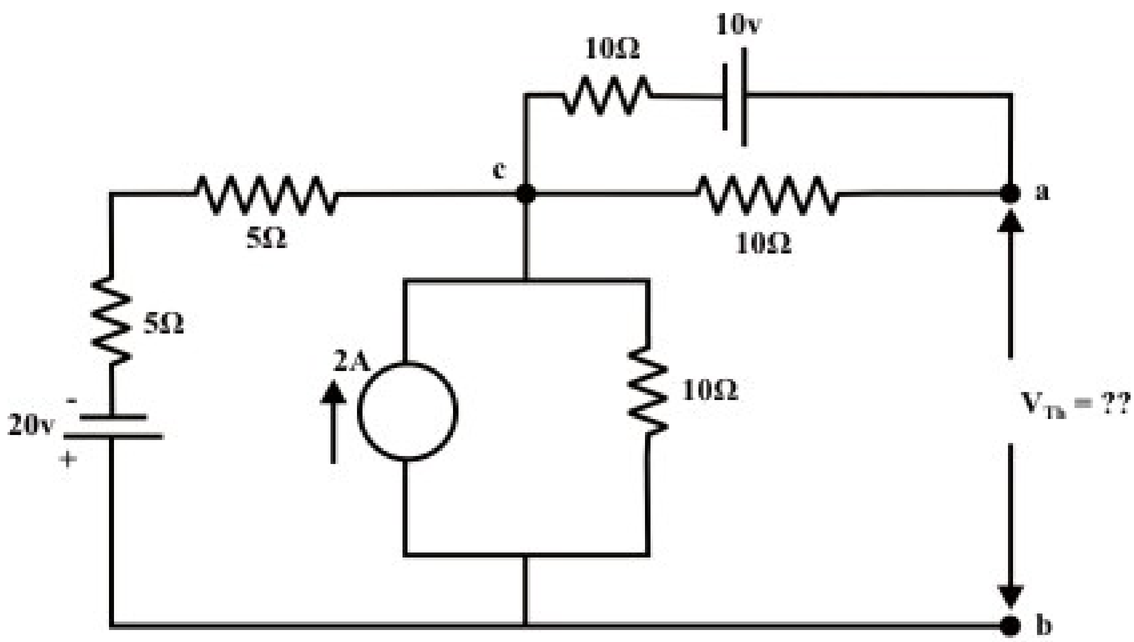


Fig. 8.7(b)

The above circuit is equivalently represented by a Thevenin circuit and the corresponding Thevenin voltage V_{Th} and Thevenin resistance R_{Th} are calculated by following the steps given below:

Now applying 'Super position theorem', one can find V_{Th} (voltage across the 'a' and 'b' terminals, refer fig. 8.7(b)). Note any method (node or mesh analysis) can be applied to find V_{Th}.

Considering only 20v source only

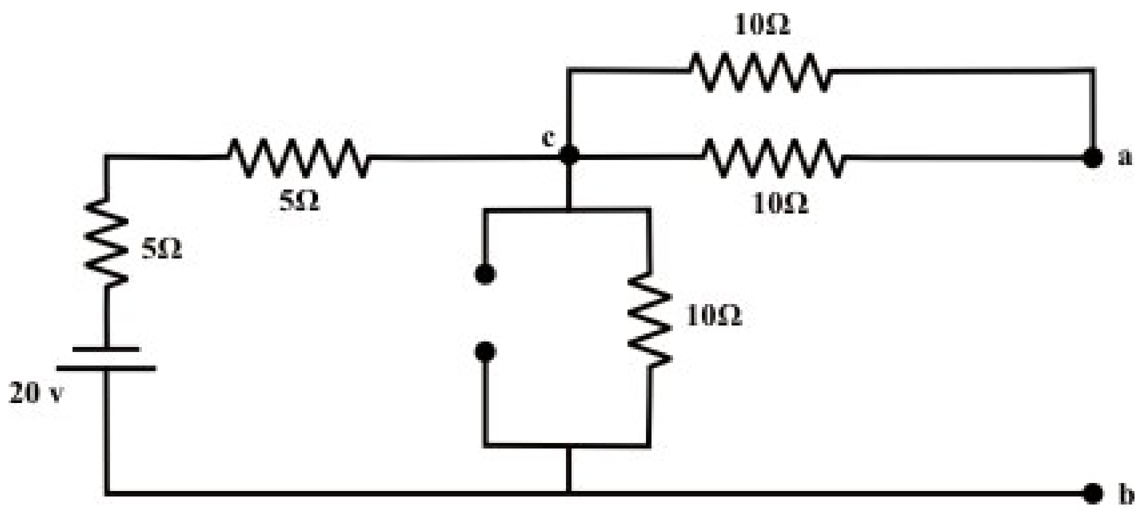


Fig. 8.7(c)

From the above circuit the current through 'b-c' branch $= \frac{20}{20} = 1A$ (from 'b' to 'a') whereas the voltage across the 'b-a' branch $v_{ba} = 1 \times 10 = 10\ volt$. ('b' is higher potential than 'a'). $\therefore v_{ab} = -10\, volt$

Considering only 10v source only

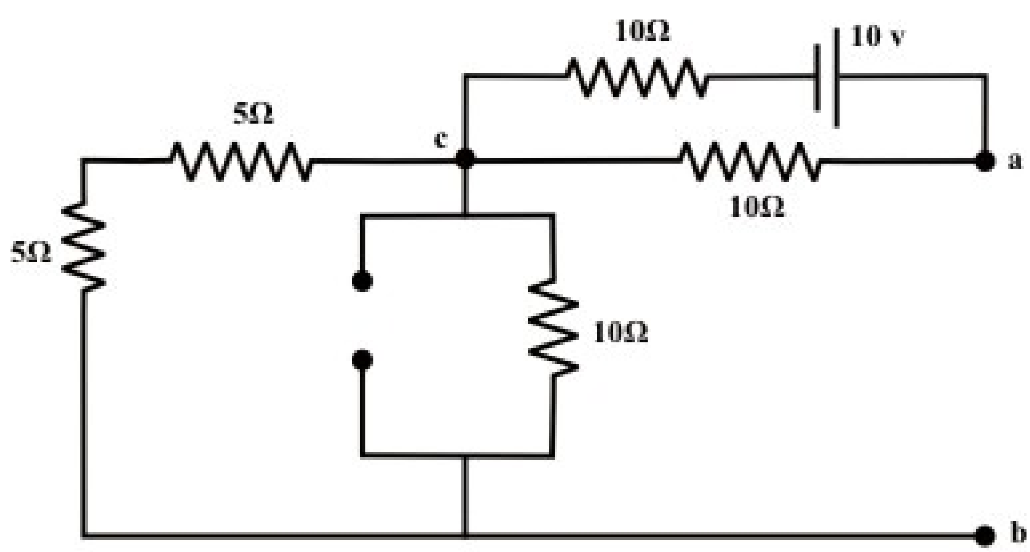


Fig. 8.7(d)

Note: No current is flowing through 'cb'-branch.
$\therefore$ V_{ab} = 5v ('a' is higher potential than 'b')

Consider only $2\,A$ current source only

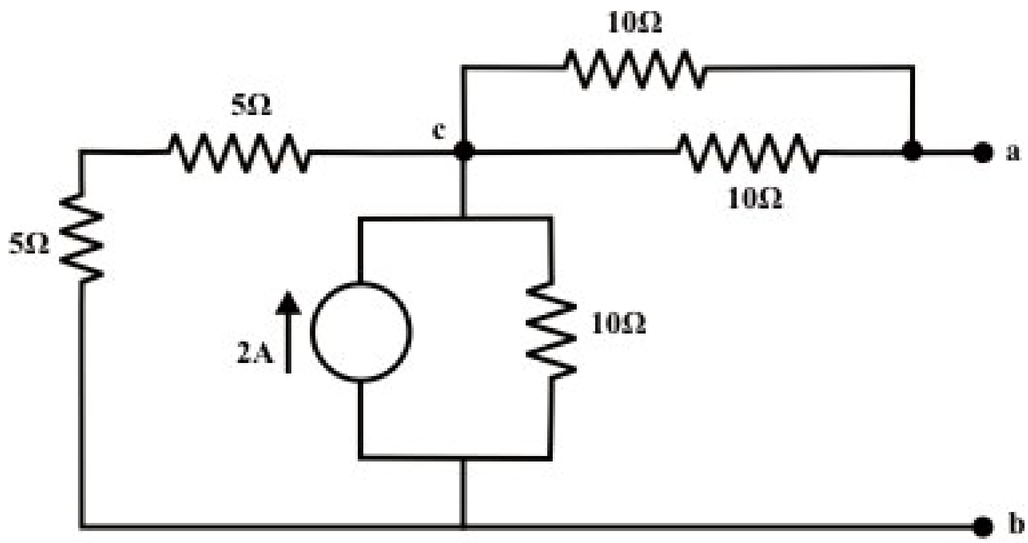


Fig. 8.7(e)

Note that the current flowing the 'c-a' branch is zero
$\therefore$ V_{ab} =10 v ('a' is higher potential than 'b' point).

The voltage across the 'a' and 'b' terminals due to the all sources = V_{Th} = V_{ab} (due to 20v) + V_{ab} (due to 10v) + V_{ab} (due to 2A source) = - 10 + 5 + 10 = 5v (a is higher potential than the point 'b').

To computute R_{Th}:

Replace all voltage and current sources by their internal resistance of the circuit shown in fig.8.7(b).

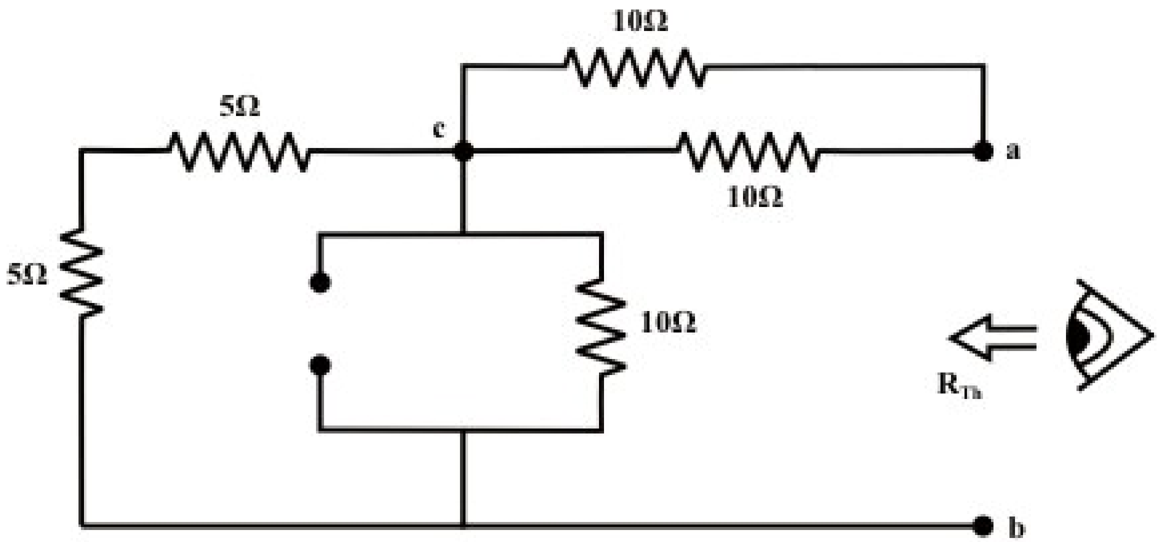
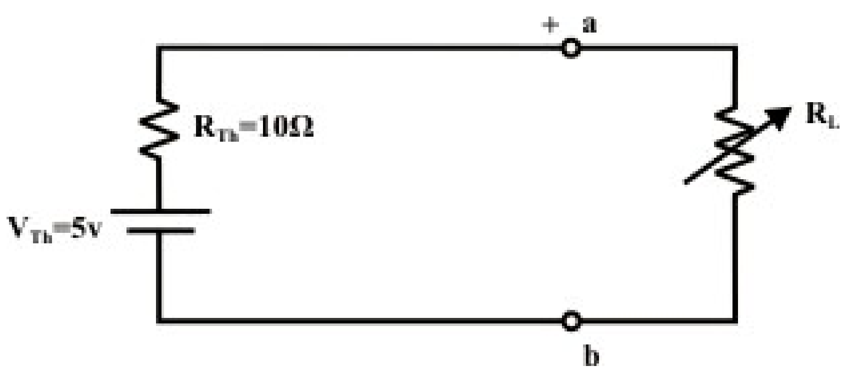


Fig. 8.7(f)

$R_{Th} = R_{ab} = ((5+5) \parallel 10) + (10 \parallel 10)$
$= 5 + 5 = 10\ \Omega$

Thevenin equivalent circuit is drawn below:



The choice of R_L that absorbs maximum power from the circuit is equal to the value of Thevenin resistance R_{Th}

$R_L = R_{Th} = 10\Omega$

Under this condition, the maximum power dissipated to R_L is

$$P_{max} = \frac{1}{4}\frac{V_{Th}^2}{R_{Th}} = \frac{1}{4}\cdot\frac{25}{10} = 0.625 \text{ watts.}$$

L.8.5 Proof of Thevenin Theorem

The basic concept of this theorem and its proof are based on the principle of superposition theorem. Let us consider a linear system in fig.L.8.8(a).

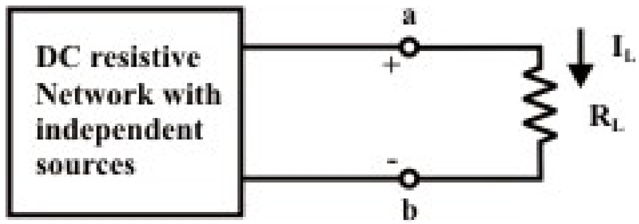


Fig. 8.8(a)

It is assumed that the dc resistive network is excited by the independent voltage and current sources. In general, there will be certain potential difference ($V_{oc} = V_{Th}$) between the terminals 'a' and 'b' when the load resistance R_L is disconnected from the terminals. Fig.8.8(b) shows an additional voltage source E (ideal) is connected in series with the load resistance R_L in such a way (polarity of external voltage source E in opposition the open-circuit voltage V_{oc} across 'a' and 'b' terminals) so that the combined effect of all internal and external sources results zero current through the load resistance R_L.

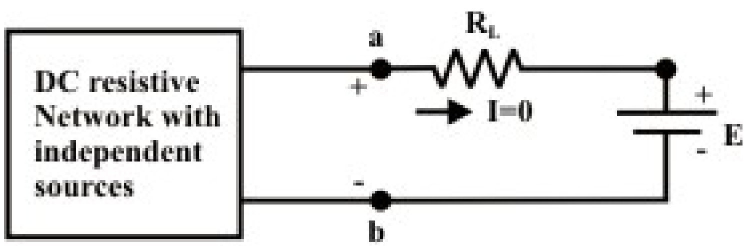


Fig. 8.8(b)

According to the principle of superposition, zero current flowing through R_L can be considered as a algebraic sum (considering direction of currents through R_L) of (i) current through R_L due to the external source E only while all other internal sources are replaced by their internal resistances (all voltage sources are short-circuited and all current sources are open circuited), and (ii) current through R_L due to the combined effect of all internal sources while the external source E is shorted with a wire. For the first case, assume the current $I_1 \left(= \dfrac{E}{R_{Th} + R_L} \right)$ (due to external source E only) is flowing through R_L from right to left direction($\leftarrow$) as shown in fig.8.8(c).

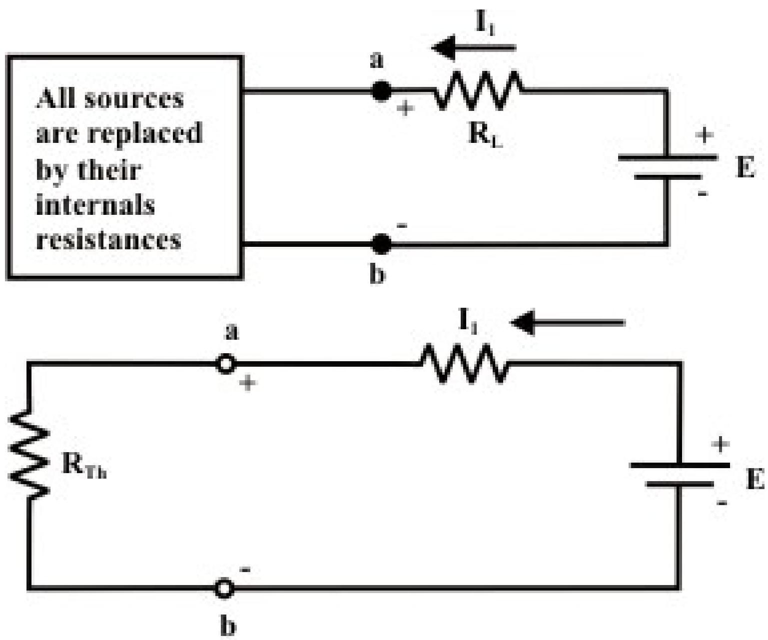


Fig. 8.8(c)

For the second case, the current I_2 (due to combined effect of all internal sources only) is flowing through R_L with same magnitude of I_1 but in opposite direction (left to right). Note that the resultant current I through the resistor R_L is zero due to the combination of internal and external sources (see fig.8.8(b)). This situation will arise provided the voltage (V_{ab}) across the 'a' and 'b' terminals is exactly same (with same polarity) as that of external voltage E and this further implies that the voltage across V_{ab} is nothing but an open-circuit voltage (V_{Th}) while the load resistance R_L is disconnected from the terminals 'a' and 'b'. With the logics as stated above, one can find the current expression $I_2\left(=\dfrac{V_{Th}}{R_{Th}+R_L}\right)$ for the circuit (fig.8.8(b)) when the external source E is short-circuited with a wire. In other words, the original circuit (fig.8.8(a)) can be replaced by an equivalent circuit that delivers the same amount of current I_L through R_L. Fig.8.8(d) shows the equivalent Thevenin circuit of the original network (fig.8.8(a)).

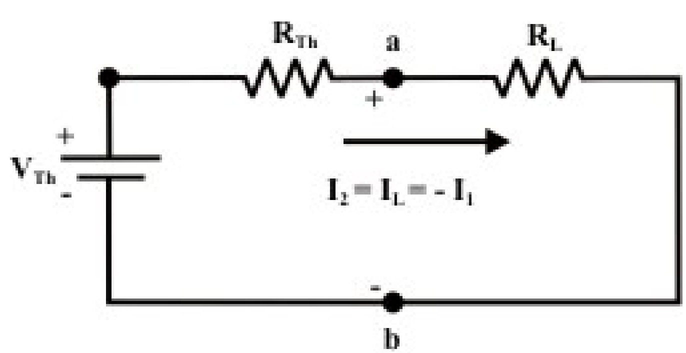


Fig. 8.8(d)

L.8.6 Norton's theorem

Norton's theorem states that any two terminals A & B of a network composed of linear resistances (see fig.8.9(a)) and independent sources (voltage or current, combination of voltage and current sources) may be replaced by an equivalent current source and a parallel resistance. The magnitude of current source is the current measured in the short circuit placed across the terminal pair A & B. The parallel resistance is the equivalent resistance looking into the terminal pair A & B with all independent sources has been replaced by their internal resistances.

Any linear dc circuit, no matter how complicated, can also be replaced by an equivalent circuit consisting of one dc current source in parallel with one resistance. Precisely, Norton's theorem is a dual of Thevenin's theorem. To find a current I_L through the load resistance R_L (as shown in fig.8.9(a)) using Norton's theorem, the following steps are followed:

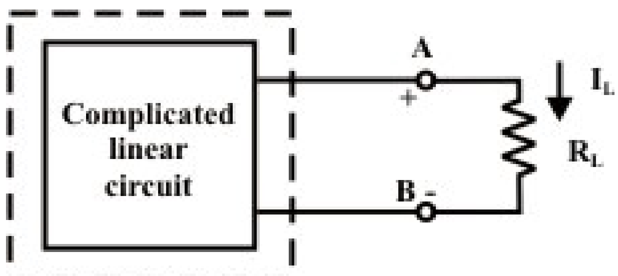


Fig. 8.9(a)

Step-1: Short the output terminal after disconnecting the load resistance (R_L) from the terminals A & B and then calculate the short circuit current I_N (as shown in fig.8.9(b)). In general, one can apply any of the techniques (mesh-current, node-voltage and superposition method) learnt in earlier lessons to compute I_N (experimentally just measure the short-circuit current using an ammeter).

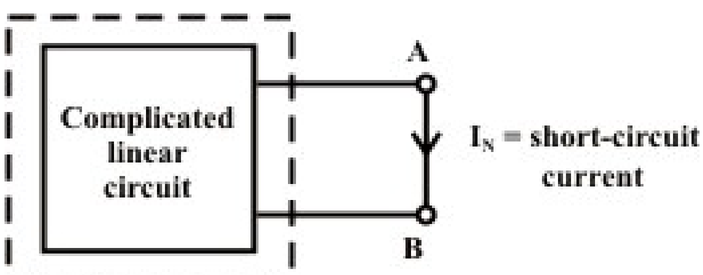


Fig. 8.9(b)

Step-2: Redraw the circuit with each practical sources replaced by its internal resistance while the short–circuit across the output terminals removed (note: voltage sources should be short-circuited (just replace with plain wire) and current sources should be open-circuited (just removed)). Look backward into the resulting circuit from the load terminals ($A \& B$), as suggested by the eye in fig.8.9(c).

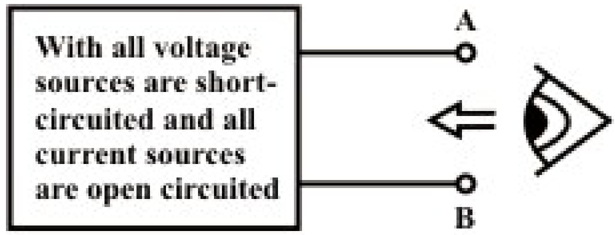


Fig. 8.9(c)

Step-3: Calculate the resistance that would exist between the load terminals $A \ \& \ B$ (or equivalently one can think as if a voltage source is applied across the load terminals and then trace the current distribution through the circuit (fig.8.9(c)) in order to calculate the resistance across the load terminals). This resistance is denoted as R_N, is shown in fig.8.9 (d). Once again, calculating this resistance may be a difficult task but one can try to use the standard circuit reduction technique or $Y-\Delta$ *or* $\Delta-Y$ transformation techniques. It may be noted that the value of Norton's resistance R_N is truly same as that of Thevenin's resistance R_{Th} in a circuit.

Step-4: Place R_N in parallel with current I_N to form the Norton's equivalent circuit (replacing the imaginary fencing portion or fixed part of the circuit with an equivalent practical current source) as shown in fig.8.8 (d).

Step-5: Reconnect the original load to the Norton current circuit; the load's voltage, current and power may be calculated by a simple arithmetic operation only.

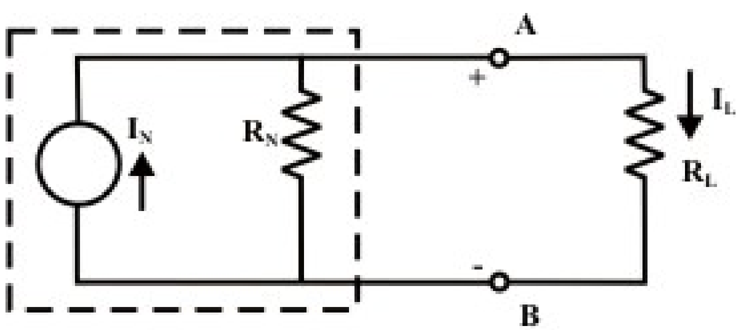


Fig. 8.9(d): Norton's equivalent circuit of the original network

Load current $I_L = \dfrac{R_N}{R_N + R_L} \times I_N$ (8.9)

Voltage across the load $V_L = I_L \times R_L$ (8.10)

Power absorbed by the load $P_L = I_L^{\,2} \times R_L$ (8.11)

Remarks: (i) Similar to the Thevenin's theorem, Norton's theorem has also a similar advantage over the normal circuit reduction technique or any other technique when it is used to calculate load current (I_L), load voltage (V_L) and load power (P_L) for different loads.

(ii)Fortunately, with help of either Norton's theorem or Thevenin's theorem one can find the choice of load resistance R_L that results in the maximum power transfer to the load.

(iii) Norton's current source may be replaced by an equivalent Thevenin's voltage source as shown in fig.L.8.1(b). The magnitude of voltage source (V_{Th}) and its internal resistances (R_{Th}) are expressed by the following relations

$V_{Th} = I_N \times R_N$; $R_{Th} = R_N$ (with proper polarities at the terminals)

In other words, a source transformation converts a Thevenin equivalent circuit into a Norton equivalent circuit or vice-verse.

L.8.7 Application of Norton's Theorem

Example-L.8.5 For the circuit shown in fig.8.10(a), find the current through $R_L = R_2 = 1\Omega$ resistor (I_{a-b} branch) using Norton's theorem & hence calculate the voltage across the current source (V_{cg}).

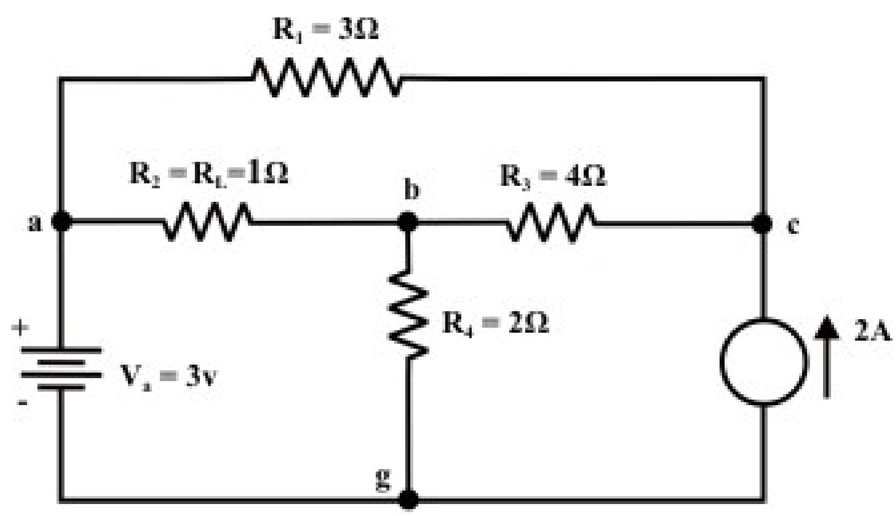


Fig. 8.10(a)

Solution:

Step-1: Remove the resistor through which the current is to be found and short the terminals 'a' and 'b' (see fig.8.10(b)).

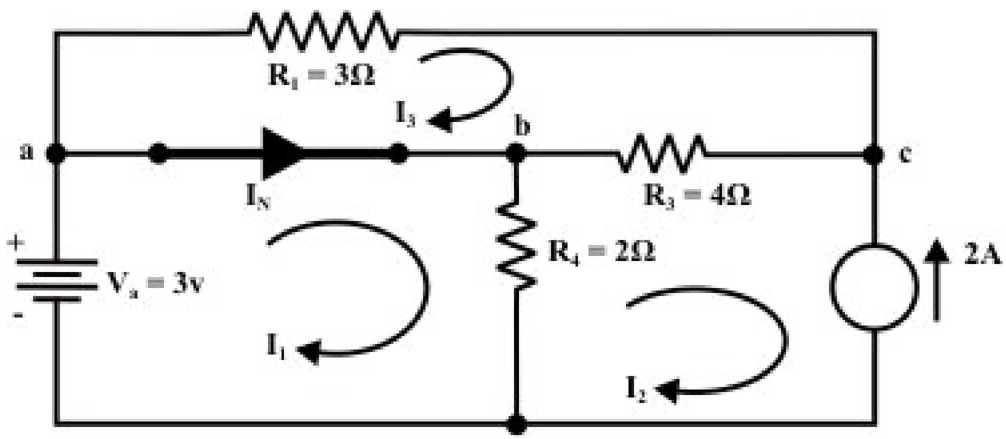


Fig. 8.10(b)

Step-2: Any method can be adopted to compute the current flowing through the a-b branch. Here, we apply 'mesh – current' method.

Loop-1

$3 - R_4(I_1 - I_2) = 0$, where $I_2 = -2A$

$R_4 I_1 = 3 + R_4 I_2 = 3 - 2 \times 2 = -1 \quad \therefore I_1 = -0.5A$

Loop-3

$$-R_1 I_3 - R_3(I_3 - I_2) = 0$$
$$-3I_3 - 4(I_3 + 2) = 0$$
$$-7I_3 - 8 = 0$$
$$I_3 = -\frac{8}{7} =$$
$$\therefore I_N = (I_1 - I_3) = \left(-0.5 + \frac{8}{7}\right) = \frac{-7 + 16}{14}$$
$$= \frac{9}{14}\text{A (current is flowing from 'a' to 'b')}$$

Step-3: To compute R_N, all sources are replaced with their internal resistances. The equivalent resistance between 'a' and 'b' terminals is same as the value of Thevenin's resistance of the circuit shown in fig.8.3(d).

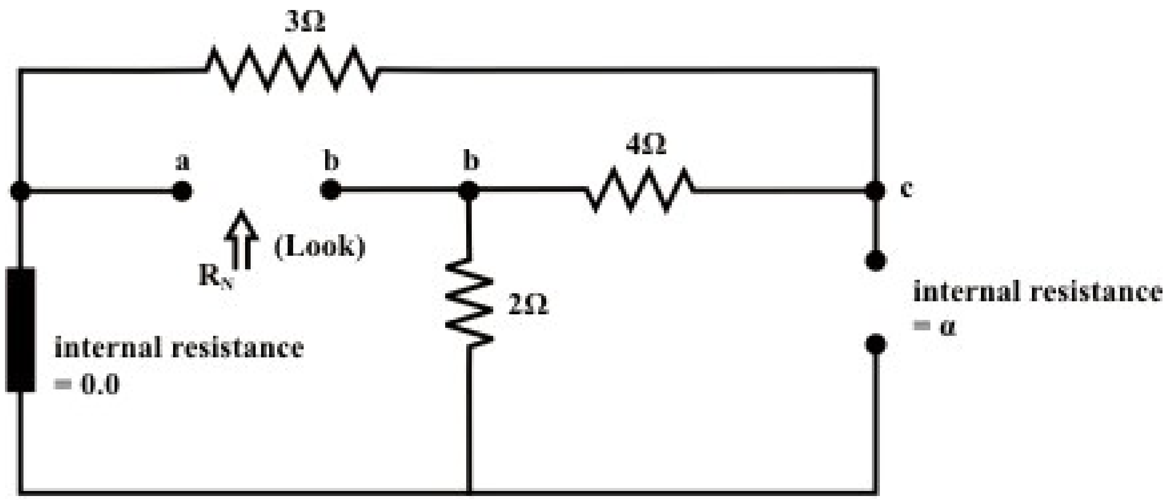


Fig. 8.10(c)

Step-4: Replace the original circuit with an equivalent Norton's circuit as shown in fig.8.10(d).

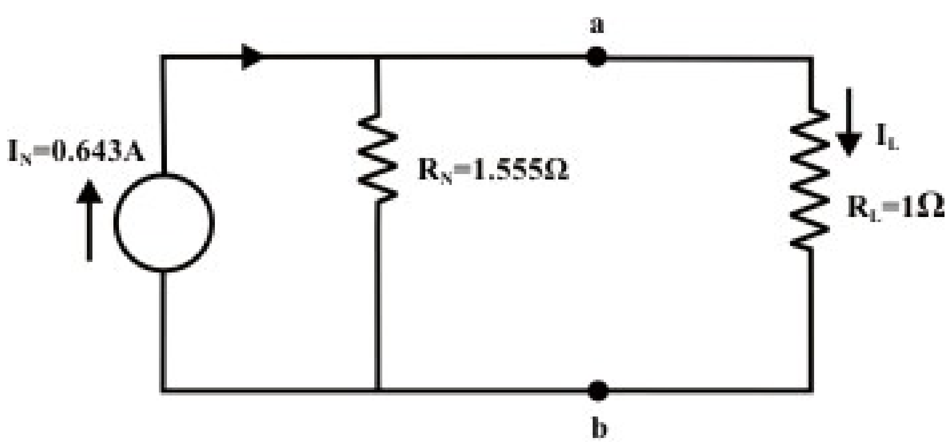
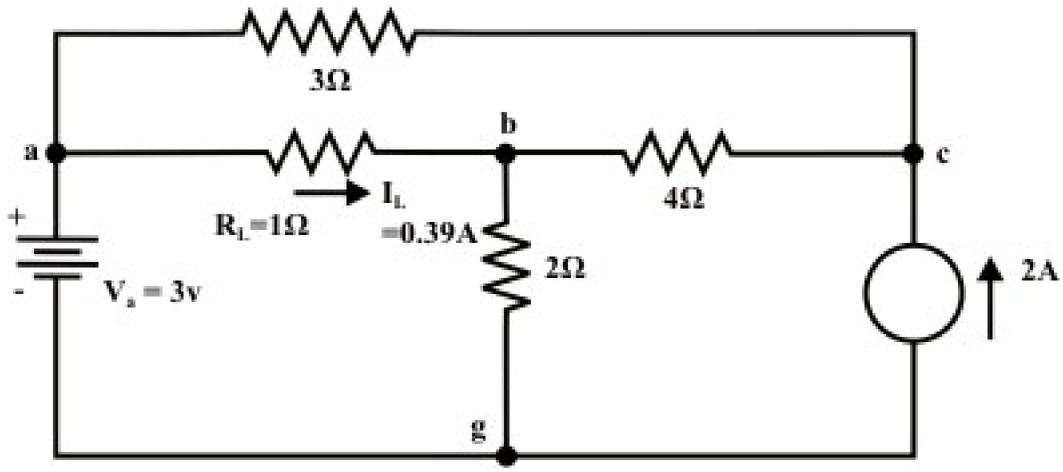


Fig. 8.10(d)

$$I_L = \frac{R_N}{R_N + R_L} \times I_N = \frac{1.555}{1.555+1} \times 0.643 = 0.39\text{A (a to b)}$$

In order to calculate the voltage across the current source the following procedures are adopted. Redraw the original circuit indicating the current direction in the load.



$$V_{bg} = 3 - 1\times 0.39 = 2.61\text{volt}$$

$$I_{bg} = \frac{2.61}{2} = 1.305\text{A}$$

$$I_{cb} = 1.305 - 0.39 = 0.915\text{A ('c' to 'b')}$$

$$\therefore\ V_{cg} = 2\times 1.305 + 4\times .915 = 6.26\text{volt ('c' is higher potential than 'g')}$$

Example-L.8.6 For the circuit shown in fig.8.11(a), the following measurements are taken and they are given in table.

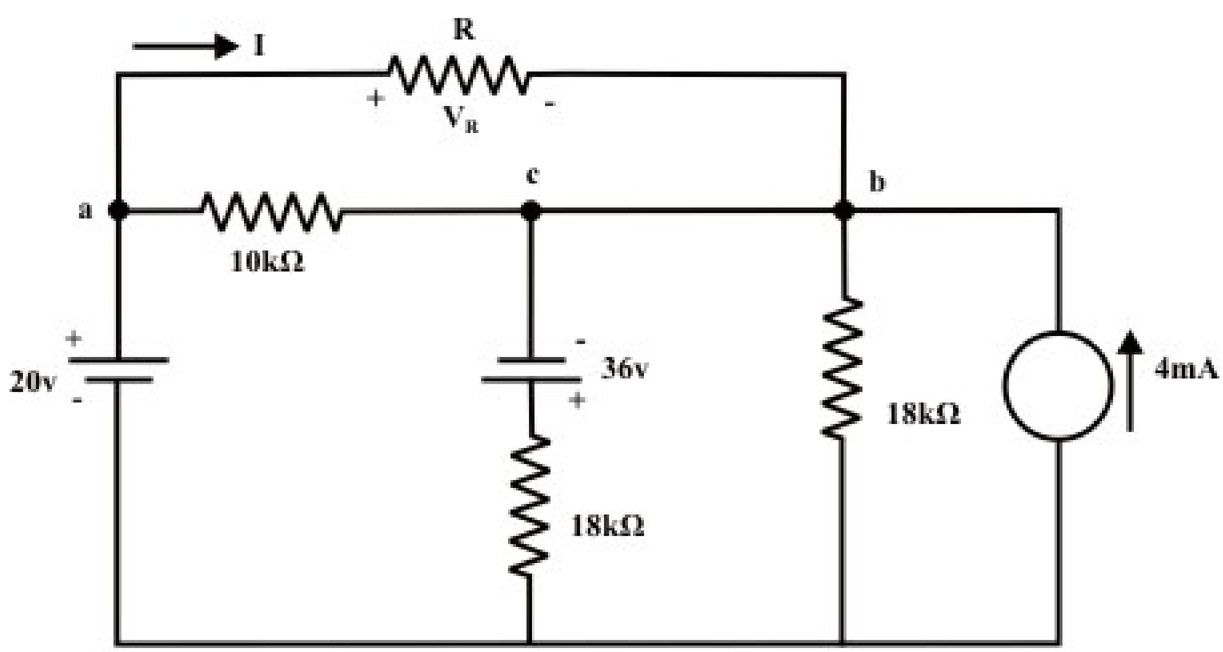


Fig. 8.11(a)

Table

R	I	V_R
Open	$0\ ma$	$1.053\,V$
Short	$0.222\,ma$	$0V$
?	$0.108\ ma$	**?**
$25k\Omega$	**?**	**?**

Find the current following through the resistor when $R = 25k\Omega$ and voltage drop across the resistor.

Solution: First measurement implies the Thevenin's voltage (V_{Th}) across the terminals 'a' and 'b' $= 1.053\,V$.

Second measurement implies the Norton's current (I_N) through the shorted terminals 'a' and 'b' $= 0.222\,ma$.

With the above two measurements one can find out the Thevenin's resistance $R_{Th}\,(= R_N)$ using the following relation

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{1.053}{0.222\times10^{-3}} = 4.74\,k\Omega$$

Thevenin equivalent circuit between the terminals 'a' and 'b' of the original circuit is shown in fig.8.11(b).

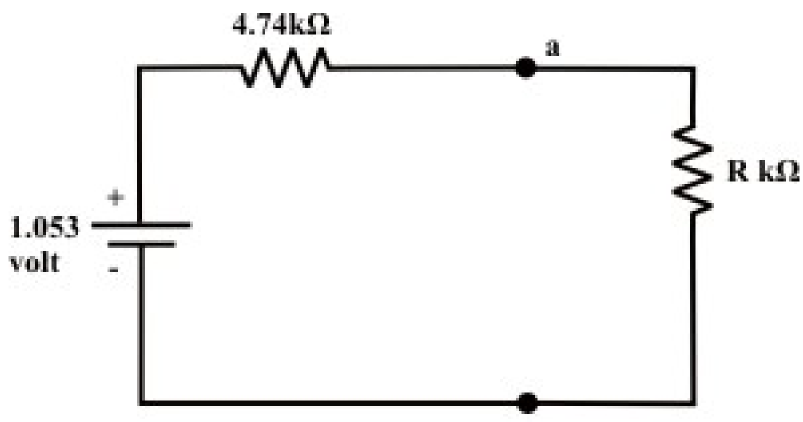


Fig. 8.11(b): Therenin equivalent circuit between the terminal 'a' and 'b'

Third measurement shows that the current in resistor R is given by $= \dfrac{1.053}{4.74 + R} = 0.108\,(mA) \;\Rightarrow\; R = 5\,k\,\Omega$. The voltage across the $5k\,\Omega$ resistor is $5\times0.108\,=\,0.54\,volt$

From the fourth measurement data, the current through $25k\Omega$ resistor is = $I = \frac{V_{Th}}{R_{Th}+R} = \frac{1.053}{(4.74+25)\times 10^3} = 0.0354\ ma$ and the corresponding voltage across the resistor $V_R = I \times R = 0.0354 \times 25 = 0.889V$.

Example-L.8.7 Applying Norton's theorem, calculate the value of R that results in maximum power transfer to the 6.2Ω resistor in fig.8.12(a). Find the maximum power dissipated by the resistor 6.2Ω under that situation.

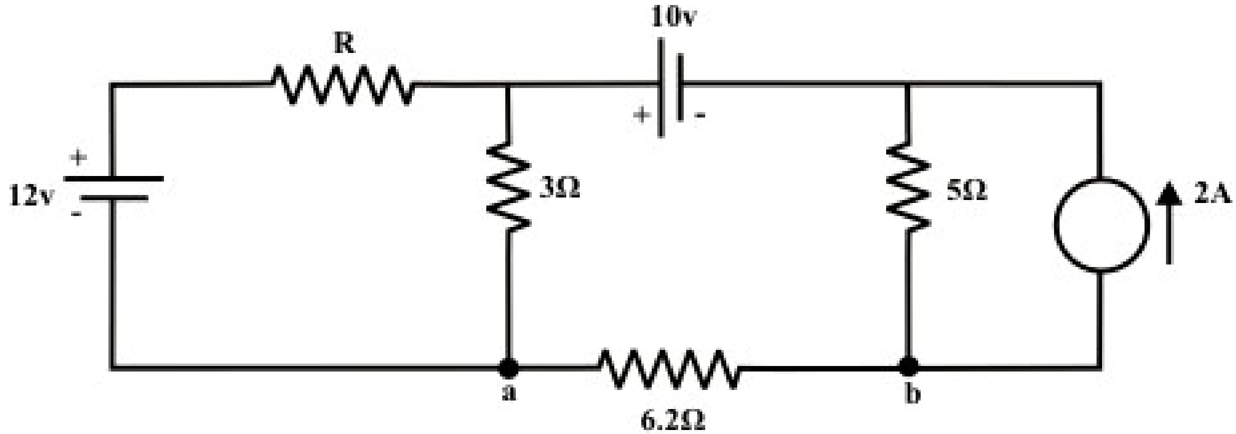


Fig. 8.12(a)

Solution:

Step-1: Short the terminals 'a' and 'b' after disconnecting the 6.2Ω resistor. The Norton's current I_N for the circuit shown in fig.8.12(b) is computed by using 'mesh-current' method.

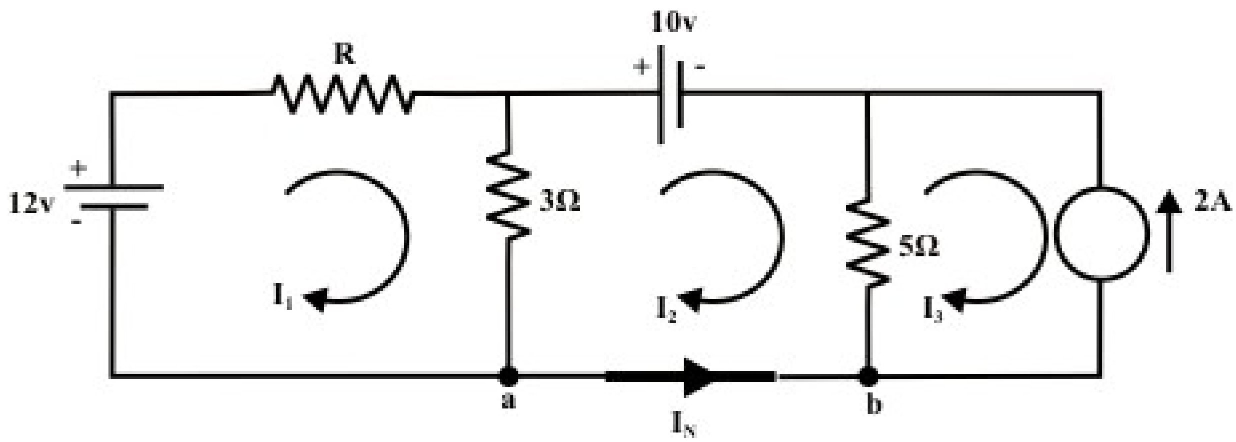


Fig. 8.12(b)

Loop-1:

$$12 - I_1 R - 3(I_1 - I_2) = 0 \tag{8.12}$$

Loop-2:

$$-10 - 5(I_2 - I_3) - 3(I_2 - I_1) = 0\ , \quad note\ \ I_3 = -2\,A \tag{8.14}$$

Solving equations (8.12) and (8.14), we get
$I_1 = \dfrac{36}{15+8R}$; $I_2 = -\dfrac{24+20R}{15+8R}$ (-ve sign implies that the current is flowing from 'b' to 'a') and Norton's current $I_N = -I_2 = \dfrac{24+20R}{15+8R}$

Norton's resistance R_N is computed by replacing all sources by their internal resistances while the short-circuit across the output terminal 'a' and 'b' is removed. From the circuit diagram fig.8.12(c), the Norton's resistance is obtained between the terminals 'a' and 'b'.

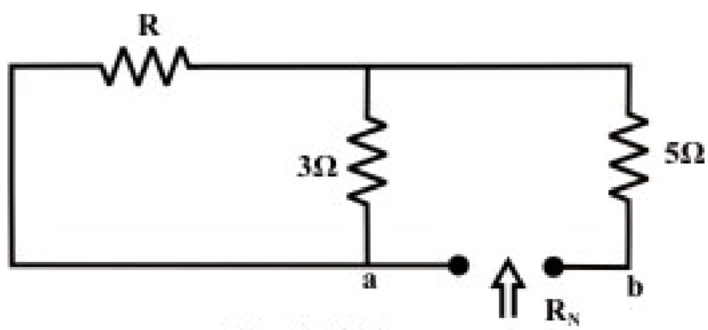


Fig. 8.12(c)

$$R_N = (R \| 3) + 5 = \frac{3R}{3+R} + 5 \tag{8.15}$$

Note that the maximum power will dissipate in load resistance when load resistance = Norton's resistance $R_N = R_L = 6.2\,\Omega$. To satisfy this condition the value of the resistance R can be obtained from equation (8.15), we get $R = 2\,\Omega.$. The circuit shown in fig.8.12(a) is now replaced by an equivalent Norton's current source (as shown in fig.L.8.12(d)) and the maximum power delivered by the given network to the load $R_L = 6.2\,\Omega$ is thus given by

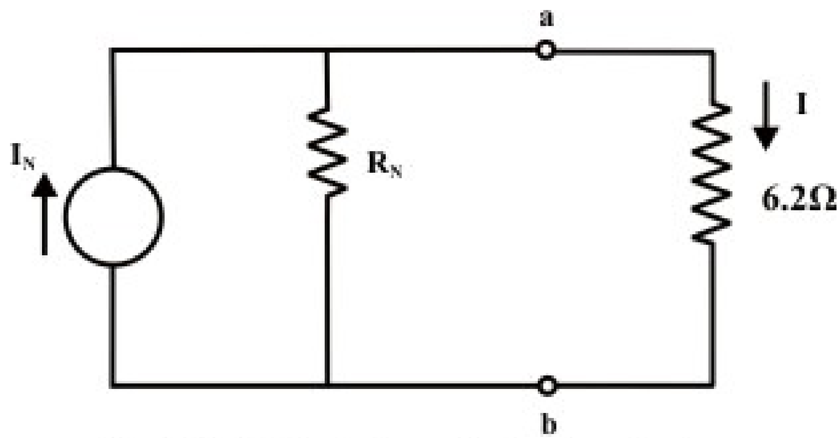


Fig. 8.12(d): Norton's equivalent circuit of original circuit

$$P_{max} = \frac{1}{4} \times I_N^{\ 2} R_L = \frac{1}{4} \times \left(\frac{24 + 20R}{15 + 8R} \right)^2 \times R_L = 6.61 watts$$

L.8.8 Test Your Understanding [Marks: 60]

T.1 When a complicated dc circuit is replaced by a Thevenin equivalent circuit, it consists of one ------- in series with one --------- . [2]

T.2 When a complicated dc circuit is replaced by a Norton equivalent circuit, it consists of ------ in ----- with one -------. [2]

T.3 The dual of a voltage source is a -----------. [1]

T.4 When a Thevenin theorem is applied to a network containing a current source; the current source is eliminated by --------- it. [1]

T.5 When applying Norton's theorem, the Norton current is determined with the output terminals --------------, but the Norton resistance is found with the output terminals ---------.and subsequently all the independent sources are replaced -----------. [3]

T.6 For a complicated circuit, the Thevenin resistance is found by the ratio of -------- voltage and ------------ current. [2]

T.7 A network delivers maximum power to the load when its -------- is equal to the -------- of circuit at the output terminals. [2]

T.8 The maximum power transfer condition is meaningful in ------------ and --------- systems. [2]

T.9 Under maximum power transfer conditions, the efficiency of the system is only --------- %. [1]

T.10 For the circuit in fig.8.13, find the voltage across the load resistance $R_L = 3\Omega$ using Thevenin theorem. Draw the Thevenin equivalent circuit between the terminals 'a' and 'b' when the load resistance R_L is disconnected. Calculate the maximum power delivered by the circuit to the load $R_L = 3\Omega$. [6]

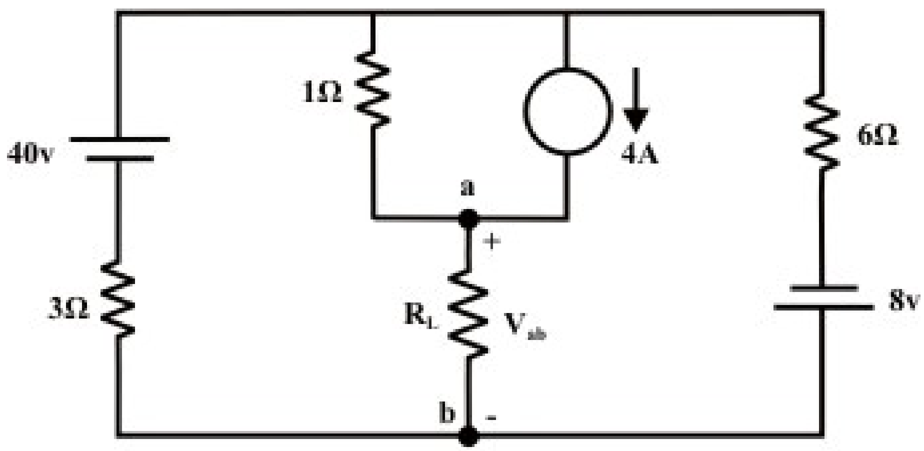


Fig. 8.13

(Ans. $V_{ab} = 18$ volt, $P_{max} = 108\, W$)

T.11 Solve the problem given in T.10 applying Norton's theorem. [6]
(Ans. $I_N = 12\, A$, $R_N = 3\Omega$)

T.12 For the circuit in fig.8.14, calculate the value of R that results in maximum power transfer to the 10Ω resistor connected between (i) 'a' and 'b' terminals (ii) 'a' and 'c' terminals. Indicate the current direction through (a) a-b branch (b) a-c branch and their magnitudes. [6+6]

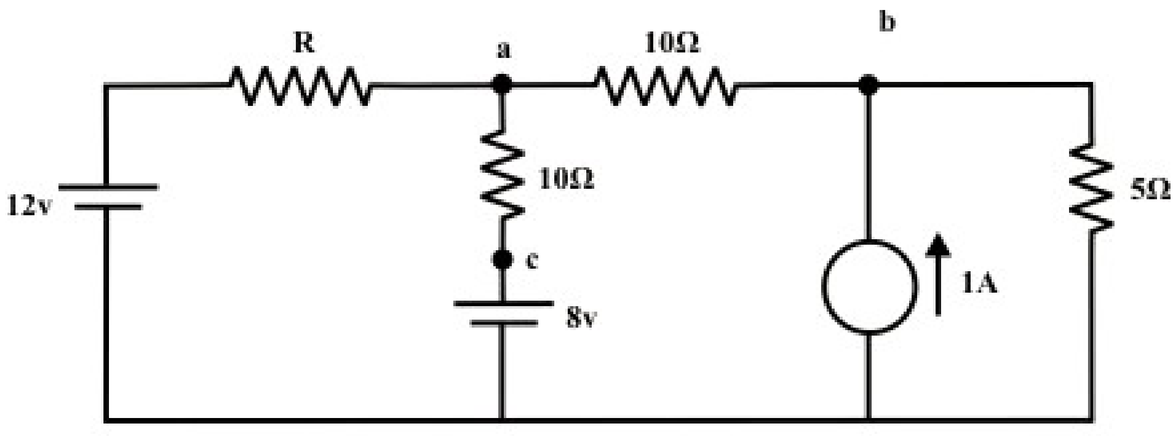


Fig. 8.14

(Ans. (i) $R = 10\Omega$, $I_{ab} = 250\, mA\ (a \to b)$ (ii) $R = 30\Omega$, $I_{ac} = 33.3\ mA\ (a \to c)$)

T.13 The box shown in fig.8.15 consists of a dc sources and resistors. Measurements are made at the terminals 'a' and 'b' and the results are shown in the table. Find the choice of 'R' that delivers maximum power to it and subsequently predict the reading of the ammeter under this situation. [6]

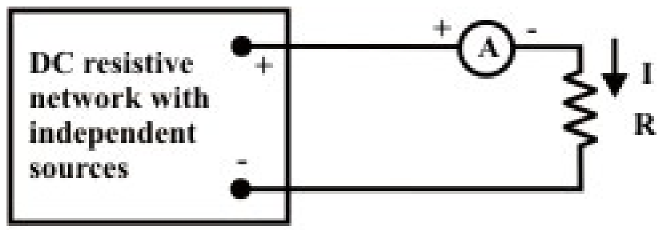


Fig. 8.15

Table

R	I
10Ω	$2\,A$
80Ω	$0.6\,A$

(Answer: $R = 20\Omega$, $P_{\max} = 45\, watts$)

T.14 For the circuit shown in fig.8.16, find the value of current I_L through the resistor $R_L = 6\Omega$ using Norton's equivalent circuit and also write the Norton's equivalent circuit parameters between the terminals $'A'$ *and* $'B'$. [7]

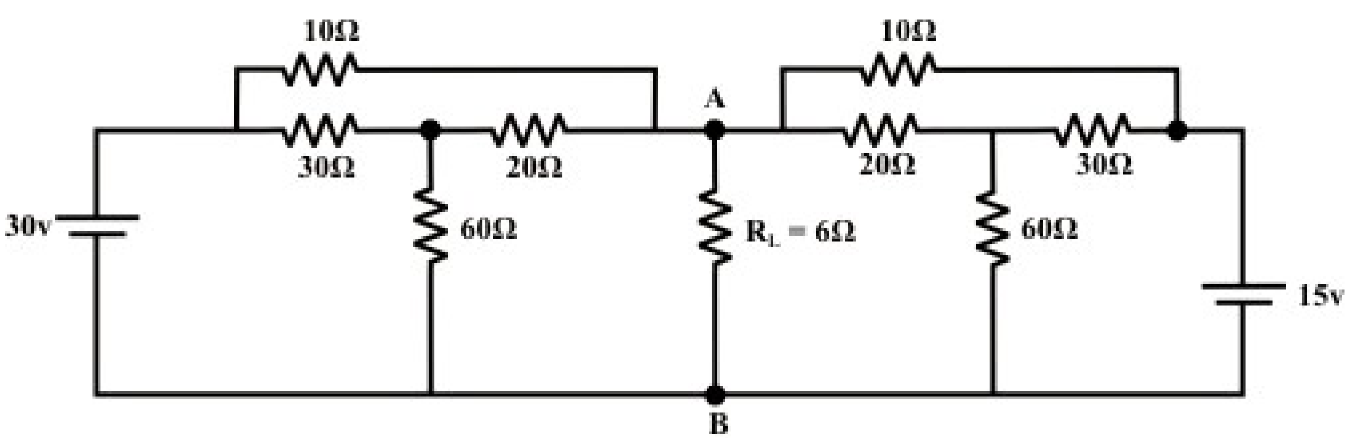


Fig. 8.16

(Ans. $I_L = 2.1\ A$, $I_N = 5.25\,A$; $R_N = 4\Omega$)

T.15 Find the values of design parameters R_1, R_2 *and* R_3 such that system shown in fig.17(a) satisfies the relation between the current I_L and the voltage V_L as described in fig.8.17(b). Assume the source voltage $V_s = 12\,volt$ and the value of resistance R_2 is the geometric mean of resistances R_1 & R_3. [7]

(Ans. $R_1 = R_2 = R_3 = 0.5 k\Omega$)

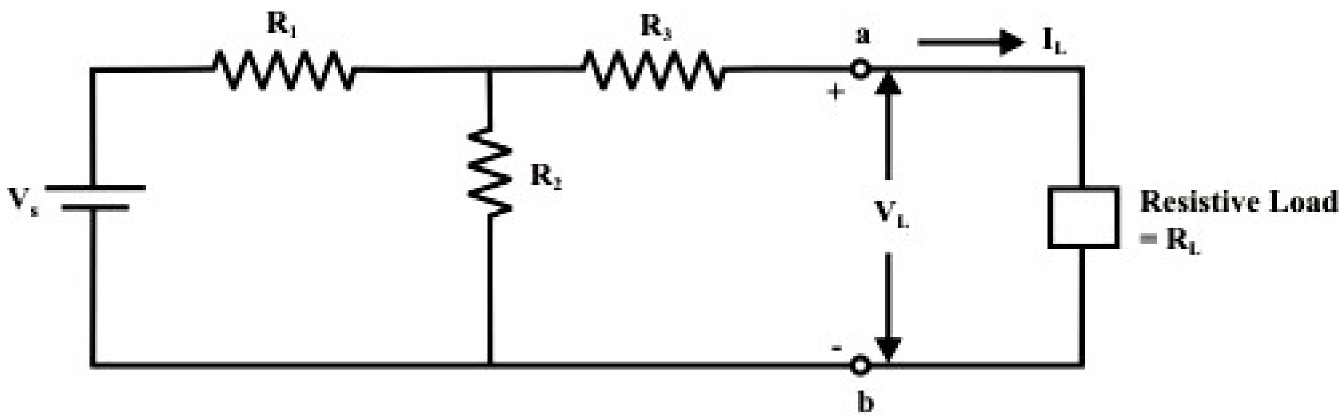


Fig. 8.17(a)

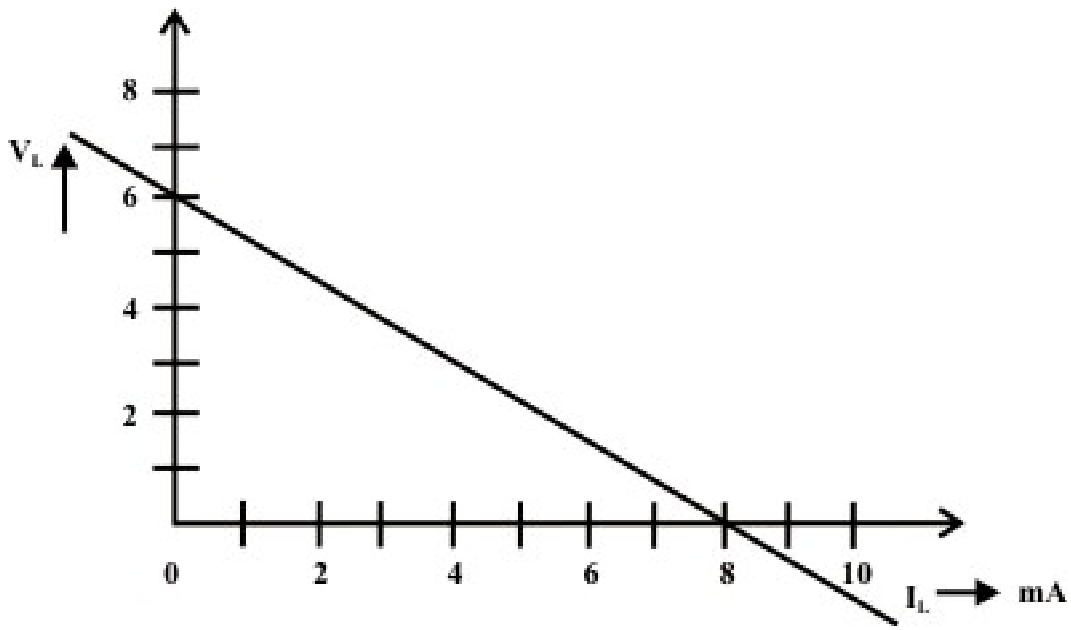


Fig. 8.17(b): Volt-Amp. characteristics at the terminals 'a' and 'b'